# 1 Basic math

In this first chapter, we walk through a number of so-called basic operations in arithmetic. These include addition, division and raising to the power. The purpose of this chapter is to prepare you for the calculations that will be done later in the book and that you will do yourself. For example, in the fifth chapter we are going to see that if you want to make a prediction, you actually have to perform some of these operations. More precisely, you can calculate a prediction by multiplying and adding numbers several times. Which numbers those are, we will see later on, but for now, let us focus the basic operations in this chapter.

Much of what is covered in this chapter you will have seen at one time or another, sometimes even at the beginning of primary school. It may or may not be a feast of recognition. Nevertheless, it is good to go through everything calmly, even if you think ‘yes, I remember that’. An important aim of this chapter is that you learn to estimate the outcome, especially when using a calculator. Look, if you have to multiply 123 by 456, it is important that you can estimate that the result must be something in the order of more than 100 times 400 is 40000. If you make a typo with your calculator, forget the 6 and multiply by 45 instead of 456, the result will be something in the order of 4000. However, your calculator is not going to point out this typing error to you because the calculator does not know what you intended to fill in. Another example is: suppose a cutback of 100 million is announced, then it is good to know what percentage this is of the total budget, also, and especially, to indicate whether the cutback is a lot or not.

In short, it is important to be able to place numbers, figures, in a context. In the second chapter, we will give much more shape to that context, but we will begin to do that in this first chapter itself.

The following arithmetical operations are dealt with successively. We are going to add and subtract (actually, the addition of negative numbers), multiply and divide (again, two complementary operations, as we shall see), and raise to the power and take roots (yes, indeed, these two belong together too). We then move on to logarithms and percentages, both useful for relating consecutive numbers (does something become more or less?), and we discuss interest rates. We will see that the latter is not so easy, even though we have to deal with it a lot in our lives. Finally, we will take a look at a special operator that makes it possible to quickly calculate the number of successes for a certain number of experiments. The chapter concludes with a number of exercises. For some of these exercises you will need a calculator.

## Numbers

Of course, we have been familiar with numbers since childhood. In primary school, we enthusiastically started with the tables of 2 and 3, and we probably all took an exit test and then went on to secondary school.

Numbers are all around us. Often, we see whole numbers, such as 1, 5, 14, or say 2439876, to name a large but randomly chosen number. These are positive integers, and then you also have negative integers, such as –1, –5 and –14. Between the positive and negative integers lies the number 0, as a kind of anchor or starting point. It can be useful to place the integers on an imaginary line, with 0 as the middle, and around it –1 and 1, and so on until …, –8, –7, –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, 8, …. When we think of temperatures, the negative numbers belong to frost. If it is freezing 3 degrees, then that means the temperature is –3 degrees. If it is 5 degrees today and –1 tomorrow, then the temperature goes down by 6 degrees; count from right to left: –1, 0, 1, 2, 3, 4, 5, that is, six steps.

In addition to whole numbers, we also know fractions. In many Dutch primary schools, this is dealt with in group 7. Here are three examples of fractions:

$$\frac{1}{2}, \frac{6}{7}, \text{ and } \frac{35}{13426}$$

These have also been chosen at random here. A fraction of two numbers as written above is an exact number. Often you will also see fractions as numbers with decimals. For example, for the first one you have

$$\frac{1}{2} = 0.5$$

Sometimes fractions cannot be written with a finite number of decimals, as shown by the following calculator result:

$$\frac{6}{7} = 0.857142857142857 1\ldots$$

We usually round this off as

$$\frac{6}{7} = 0.857$$

And think also of

$$\frac{1}{3} = 0.333$$

There are also special numbers that occur fairly frequently. For example, the square root of 2, written as $\sqrt{2}$ for which it holds that

$$\sqrt{2}\text{ times }\sqrt{2}\text{ is equal to }2.$$

We are all familiar with the number Pi, often written as the Greek letter for p, that is, $\pi$. The number $\pi$ is the ratio of the circumference of a circle to its diameter. This number also has an endless series of numbers after the dot, and the first ones are as follows:

$$\pi = 3.14159265359\ldots$$

There is also another special number, namely the base number of the exponential function (we will talk about that later in this chapter), and that number is (with the first nine decimal places)

$$e = 2.718281828$$

And perhaps you have heard of the number $i = \sqrt{-1}$ where the letter $i$ is literally the abbreviation for the word 'irrational'. In physics, this is an important number, but we will not come across it again in this book.

### Numbers in context

Numbers have different properties. This is partly due to the way in which measurements (or observations) are made. Think of a questionnaire, where the answers to questions have to be coded. For a question about someone's age, you can simply use the number of years of life. Someone aged 50 is then twice as old as someone with 25 years of life. However, if you ask someone about their preference for a colour, and the options are yellow, red and blue, then you can code the answers in different ways, for example, yellow = 1, red = 2 and blue = 3, but also yellow = 3, red = 1 and blue = 2. There is no natural order here, and then the numbers have meaning, but you cannot calculate with them. That means, in the first case, that red (2) is not twice yellow (1). In c, c, b, c, a, b, d, a, b, b, a, b and b, there is no natural order either.

There are four variants of contexts for numbers, also called scales. In ascending order of what you can do with them, they are the nominal scale, the ordinal scale, the interval scale, and the ratio scale. With nominally scaled numbers you can think of the colours above, but also for example of your student number. If your student number is 213450, for example, this says nothing about enrolment, previous education, age, gender or whatever, and neither does your fellow student with number 213451. The number actually has no meaningful meaning.

With the ordinal scale, the numbers have a little more meaning, because the order does mean something. If you are asked to put a number of movies in order of preference, you can give the most favourite movie the highest score and the least favourite the lowest score. These numbers, the scores, indicate the degree of preference, and a higher number means a larger preference. However, there is no natural zero, because you can subtract a value from each score and the order still remains. Also, multiplying or dividing the scores has no meaning.

The third type of scale is the interval scale. Think of the evaluation of a subject where you are asked whether you very much disagree (1), disagree (2), neither agree nor disagree (3), agree (4) and very much agree (5) with the statement of whether the teacher has explained the subject well. The order here means something, and there is also a zero point, namely the category (3) 'neither agree nor disagree'. However, multiplying still does not have much meaning, that is to say, agree (4) is not disagree twice (2). Another example is the temperature. You can say that today it is 10 degrees warmer than yesterday. However, you cannot say: 'It is twice as warm today as it was yesterday.'

The ratio scale is the fourth and final scale. This scale concerns numbers with which you can do the most calculations. These are also the kinds of numbers we will be working with most often in this book. Examples of numbers on the ratio scale are the number of euros collected in fines, the number of years in prison or the number of witnesses to a crime. The natural zero in these numbers is 0, and twice as many witnesses as 10 witnesses means 20 witnesses.

Now that we know a bit more about numbers, we can start discussing some of the so-called operations in arithmetic.

### Adding (+)

The most well-known operation is the addition of numbers. Of course:

$$2+2=4$$

and

$$2+45=47$$

while

$$777+777=1554$$

to give just a few examples. With the last sum, you usually work from right to left. First $7+7=14$ that gives the 4, the 10 of 14 you deal with at $10+70+70=150$. Of this, you take 50 separately, that gives the 5, and the 100 of 150 you treat with $100+700+700$, which is 1500. Together: $1500+50+4=1554$. This principle can also be applied to numbers with decimals, as in

$$0.777+0.777=1.554$$

We often use a calculator to add up multiple numbers, such as for instance the composition of a receipt at the supermarket. However, it is good to practise a little by challenging yourself and seeing if you can come up with a calculation with the same result as the one on the calculator. After all, you can always make a typing error.

Adding fractions is slightly more difficult. If you write them as numbers with decimals, you could use the calculator again, but that is not always exact. It is better to know the principle that you give the fractions in question all the same denominator. The denominator is the number below the division line, while the numerator is the number above the division line. So, if you want to add

$$\frac{1}{2}+\frac{6}{7}$$

then make sure that under the division line there is the same number, here 14 (= 2 times 7), as follows

$$\frac{1}{2}+\frac{6}{7}=\frac{7}{14}+\frac{12}{14}=\frac{19}{14}$$

It is useful to be able to do this yourself, just by writing it down on paper, because it helps you to make an assessment. Consider, for example, whether

$$\frac{1}{2}+\frac{4}{7}$$

is larger than 1. Here you can quickly see that the answer is yes because the sum is

$$\frac{1}{2}+\frac{4}{7}=\frac{7}{14}+\frac{8}{14}=\frac{15}{14}$$

However, there are plenty of cases where this cannot be seen so quickly. So, it is wise to always make your own calculations on a sheet of paper and with a pencil.


*Figure 1.1 In De Panne (Belgium) there is a sculpture with a calculation error.*

That things can sometimes go painfully wrong, even with sums, is proven by the numbers on the side of a work of art in the Belgian town of De Panne. The artist wanted to depict the numbers of the so-called Fibonacci series. This series of numbers is special because each successive number is the sum of the two preceding ones. The series starts at 0 and 1, and the next number is $0+1=1$. This is followed by $1+1=2$, $1+2=3$, $2+3=5$ and so you get the sequence

$$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, \ldots$$

and so on, endlessly. If you look at the picture in Figure 1.1, it all seems to go well until a little later in the series. The sequence is

$$377, 610, 987, 1597, 2584, 4181, \ldots$$

Wait a minute, the work of art does not say 4181 but 4541! Yes, and then everything goes wrong. Despite pressure from the city council, the artist refused to adjust the sculpture, and so there is a work of art out there with a miscalculation. Maybe it is worth taking a look at it?

### Subtracting (–)

The next arithmetical operation is subtracting, which is actually the opposite of adding. You can also say that reducing is adding by a negative number. So, for example, an operation like

$$9-6$$

can be read as

$$9+(-6)$$

Sometimes the outcome is not positive but negative, as is the case with

$$89-144=-55$$

A trick is to use parentheses, which make you realise that minus times minus is plus, and then

$$89-144=-(144-89)=-55$$

Fractional reduction uses the same principle as addition: you write the fractions so that they have the same denominator. Look, for example, at

$$\frac{2}{3}-\frac{1}{4}=\frac{8}{12}-\frac{3}{12}=\frac{5}{12}$$